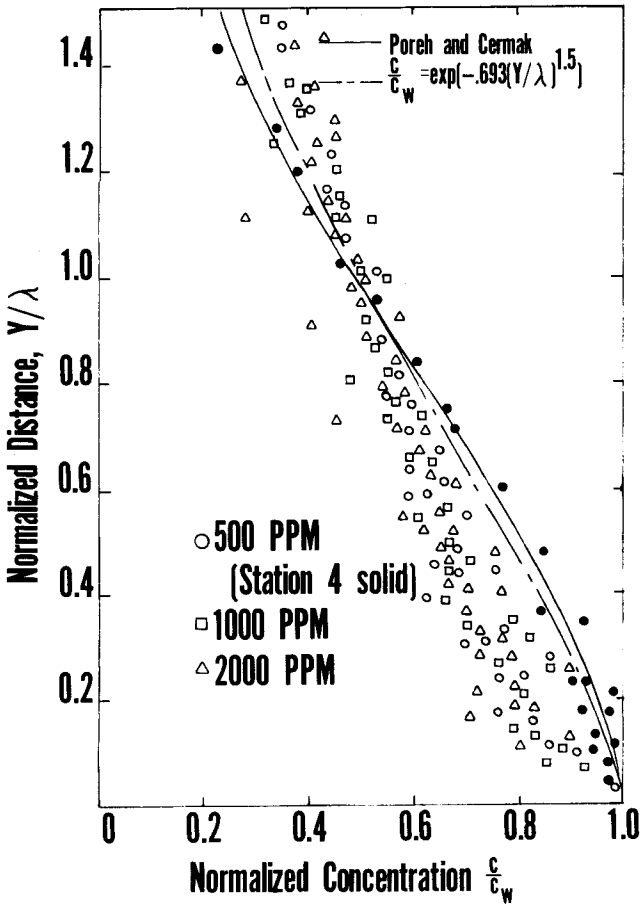

Fig. 2. Normalized concentration profile injected solution—PEO.

results show good agreement with the suggested curves for the intermediate zone at stations 2, 3, and 4. At station 1 there appears to be considerable deviation from the suggested curve. Since station 1 is in the initial zone, this deviation is expected.

The concentration profiles for injection of polymer solutions are shown in Figure 2. Only the data at station 4 for 500 ppm injection appear to be in the intermediate zone. The other data are to the left of the curves, just like station 1—water injection. For the 500 ppm injection, the concentration layer thickness at station 4 has almost reached that of the water layer. The relative rate parameter β indicates that station 4 should be in the initial zone. However, because the λ/δ ratio indicates that the diffusion has reached the intermediate zone, this is a reasonable result.

All the data for polymer injection (except as noted) deviate considerably from the intermediate zone curves and appear to be in the initial zone. The relative rate parameter β (for 1 000 and 2 000 ppm) indicates that the diffusion process should be in the intermediate zone. However, the ratio λ/δ (except for station 4 of the 1 000 ppm injection) indicates that diffusion should be in the initial zone.

Overall, the data indicate that the initial zone has been stretched out, and the growth rate of the concentration boundary layer λ is slowed by increased polymer concentration. The relative rate parameter β alone cannot characterize diffusion in the initial zone with polymer addition. PEO retards the rate of diffusion, hence, $d\lambda/dx$ decreases, and L_λ and β increase. However, normalized concentration profiles indicate the initial zone or turbulent diffusion.

NOTATION

C = polymer concentration (ppm)
C_i = polymer concentration (ppm) of injected solution
C_w = polymer concentration (ppm) at the wall
L_δ = boundary-layer growth parameter defined by Equation (1)
L_λ = concentration plume growth parameter defined by Equation (2)
x = coordinate parallel to the test surface
y = coordinate normal to the test surface
β = relative rate parameter, defined in Equation (3)
δ = boundary-layer thickness $\delta = y(0.99\ U)$
ξ = nondimensional distance $-\ \xi = y/\lambda$
λ = characteristic concentration plume height $-$ $y(0.5\ C_w)$

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Manuscript received August 21, 1975; revision received February 24, and accepted February 26, 1976.

A Model for the Residence Time Distribution of Liquid Phase in Trickle Beds

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The residence time distribution of the liquid phase has been covered by Satterfield (1975) in his excellent review on trickle-bed reactors. The earlier models included the dispersed plug flow model (Otake and Kunigita, 1958; Sater and Levenspiel, 1966; Furzer and Michell, 1970; Pham Co and Ribaud, 1971), the plug flow-stagnancy model (Hoogendoorn and Lips, 1965), the dispersed plug flow-stagnancy model (van Swaaij et al., 1969; Bennett and Goodridge, 1970), the time-delay model (Buffham et al., 1970), and the laminar film—partial mixer model (Michell and Furzer, 1972). A comparison of the models was given by Schwartz and Roberts (1973). The Michell

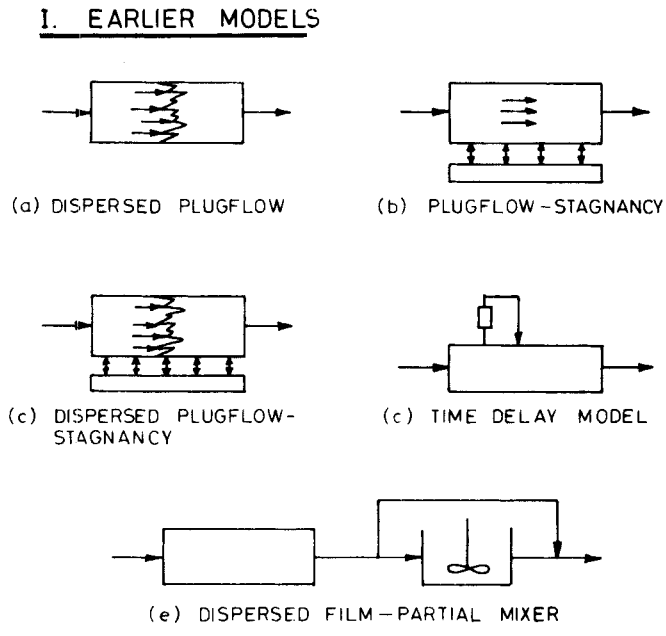

Fig. 1. The models proposed for the RTD of the liquid phase in trickle beds.

and Furzer model was an attempt to reflect the physical condition and when applied to their experimental data, predicted 90 to 94% instantaneous bypass at the packing junction, which is unlikely from the structural context of a packed bed.

THE MODEL

The present model visualizes the liquid flow as film over the surface of the particle, followed by a mixing tank corresponding to packing junction. The mixing in the film due to velocity gradients and film rippling is represented by a diffusion type of mechanism characteristic of dispersed plug flow. The mixing at the packing junction is assumed to be incomplete, with an ideally mixed zone that exchanges mass with stagnant zone. Figure 1 is schematic of the basic unit of the model, and the number of such units in series are equated to the ratio of bed height to the particle size. From the RTD expressions given by Levenspiel and Smith (1956) and by Levich et al. (1967), respectively, for dispersed film and the partial mixer, the RTD for N dispersed films is

$$E_D(\theta) = \left[\frac{Pe}{4\pi\alpha\theta}\right]^{1/2} \exp\left[-\frac{\alpha Pe(1-\theta/\alpha)^2}{4\theta}\right] \tag{1}$$

and for N nonideal mixing tanks is $(1 << N << P^{-1})$

$$E_M(\theta) = \left[\frac{N(1-PN)}{\sqrt{2\pi N}\,(1-\alpha)(1-\beta)}\right]$$

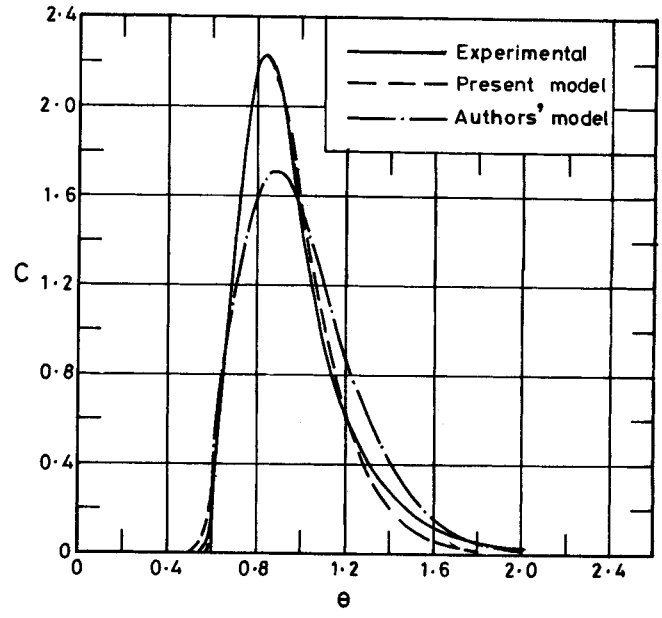

Fig. 2. Comparison of the model with the plug flow-stagnancy model and the experimental data of Hoogendoorn and Lips (1965).

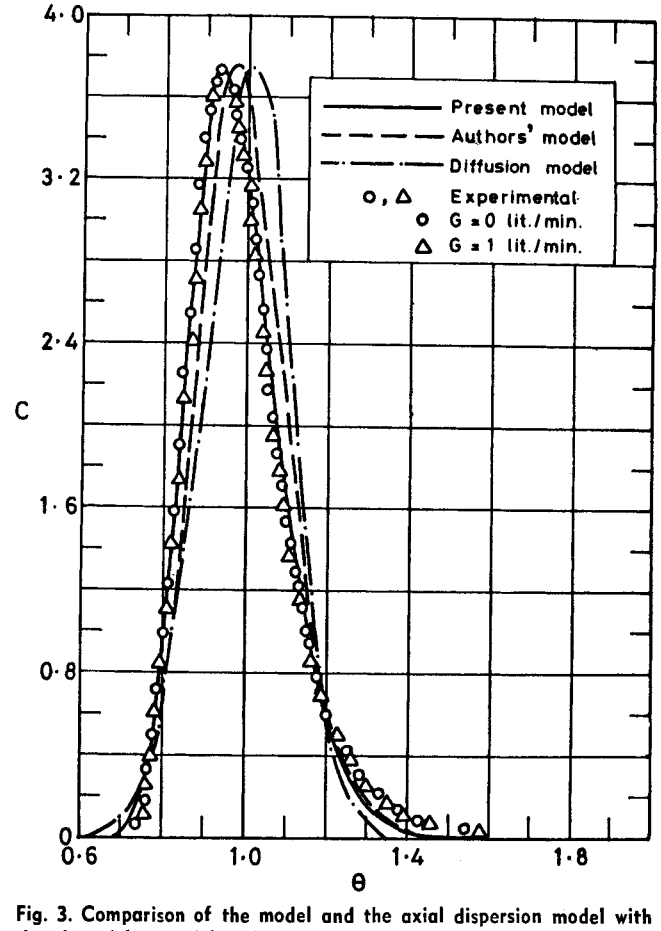

Fig. 3. Comparison of the model and the axial dispersion model with the time delay model and the experimental data of Buffham et al. (1970).

$$\exp\left[-\frac{\left\{\dfrac{\theta N}{(1-\alpha)(1-\beta)} - N\right\}^2}{2N}\right] \tag{2}$$

The total response is obtained by numerically evaluating the convolution integral

$$E(\theta) = \int_0^\theta E_D(\theta')E_M(\theta-\theta')d\theta' \tag{3}$$

TABLE 1. EXPERIMENTAL DETAILS AND EVALUATED MODEL PARAMETERS ($\alpha = 0.5$; $P = 10^{-5}$)

Experimental details					Model parameters				Reference
H	d_t	d_p	L	Re	N	Pe	Bo	β	figure
3 050	406	12.7	3.05	40.75	240	15.5	0.0646	0.28	2
3 200	38	3.2	2.62	8.75	1 500	41	0.0273	0.085	3
2 440	610	25.4	0.54	14.5	96	7.7	0.0802	0.24	4
1 830	93	9.5	1.92	19.2	200	12	0.06	0.275	5
1 830	93	9.5	4.05	40.5	200	16	0.08	0.18	5
1 830	93	9.5	6.16	61.6	200	20	0.10	0.12	5

(H: height of the column, mm; d_t: column diameter, mm; d_p: packing size, mm; L: liquid mass flow rate, Kg/m²s; α: fractional dispersed film volume; β: fractional stagnancy in mixing tank)

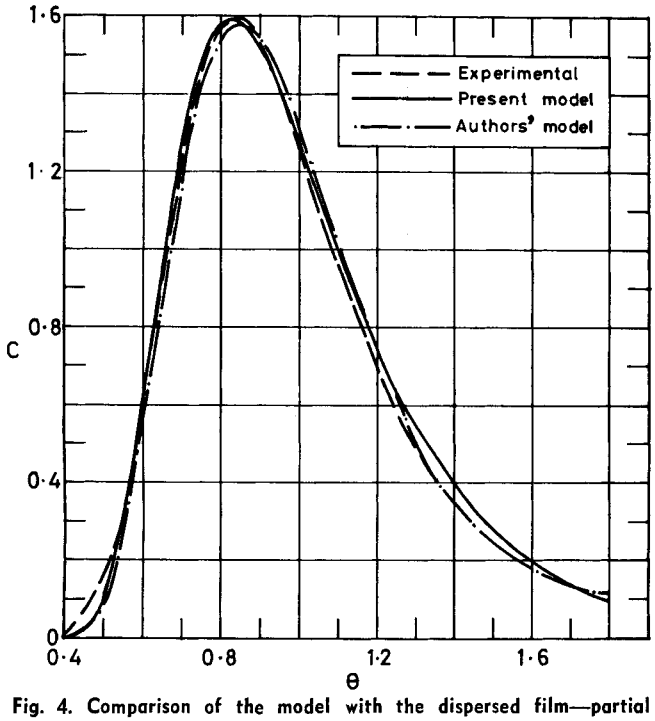

Fig. 4. Comparison of the model with the dispersed film—partial mixer model and the experimental data of Michell and Furzer (1972).

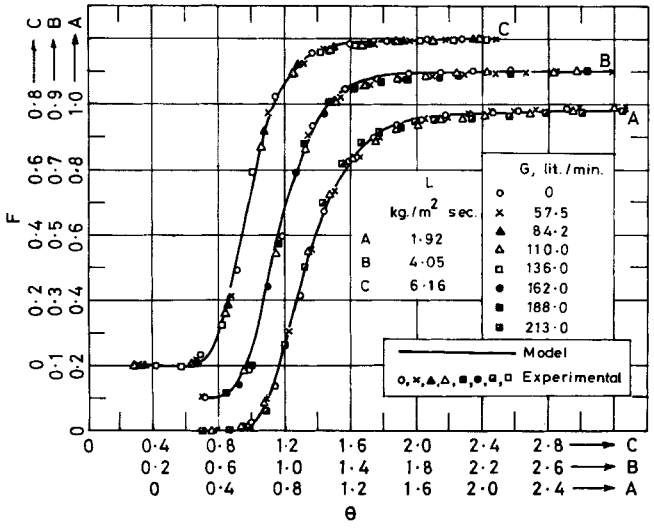

Fig. 5. Comparison of the model with the experimental data of Vasudeva (1974).

THE MODEL AND THE EXPERIMENTAL DATA

Figures 2 to 5 compare the model with the experimental data of the earlier investigators and their suggested models. Equation (3) is solved for different combinations of the variables by using an IBM 370/155 to generate a series of theoretical responses of which the curve that visually compares well with the given experimental data is chosen. α is found to vary from 0.51 to 0.47, whereas P remained constant at 10^{-5}, except for the data of Figure 5c when $P = 10^{-4}$. In view of small variation, α and P are maintained constant at 0.5 and 10^{-5}, respectively, and the response curve that matches with the experimental data with minimum root-mean-square deviation is chosen. The evaluated model parameters and the experimental conditions are listed in Table 1. The maximum root-mean-square deviation is 0.06.

A value of 0.5 for α shows that an element of liquid during its sojourn spends on an average of half its time on packing surface and the remaining period in the interstices between the packings. This is to be expected from the basic property of the dispersed film and the fact that the porosity of the packed bed is of the order of 0.5 and above. The constancy for P shows that the quantity of material (Pq) exchanging between the active and stagnant zones is proportional to the flow rate of the liquid. Thus, an increase in the flow rate leads to better mixing conditions in the packing junction which was qualitatively indicated by the earlier investigators.

The significant variables are thus the dispersion coefficient, as characterized by Bodenstein number, of the film and the fractional stagnant volume of the mixing tank. The variation in Bo and β with the liquid rate is in general agreement with the reported variation of the dispersion coefficient in the dispersed plug flow model, that Pe increases with increase in Re. The limited available experimental data, however, do not permit any relationship between Bo and β with the operating variables.

The close agreement of the model with the data of earlier investigators, obtained under different experimental conditions, proves its validity to describe liquid flow in trickle beds. The assumption of dispersed film flow over the packing surface is true of low liquid flow rates, when rippling is not predominant. The model visualizing the flow to correspond to Deans cell model at the packing junction predicts stagnancy to an extent of 30% and is thus likely to approximate the physical condition more closely than the value of 90 to 94% instantaneous bypass reported by Michell and Furzer (1972). In conclusion, the model identifies the dispersion over the packing surface and the stagnancy in the packing junction as the nonidealities and thus presents a greater insight into liquid mixing in trickle beds than the earlier models based on plug flow or dispersed plug flow with stagnancy.

NOTATION

Bo = Bodenstein number
E = residence time density function
F = residence time cumulative distribution function
G = gas flow rate, l/min
N = number of cells in the sequence
P = cross-flow ratio, defined as the fraction of feed

that exchanges between the active and stagnant regions

Pe = Peclet number

Re = liquid Reynolds number

V = volume; subscripts: p-dispersed film, m-mixing tank, s-stagnant zone, t-total

θ = dimensionless time, based on the mean holding time of the total sequence

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Manuscript received December 29, 1975; revision received March 1, and accepted March 5, 1975.

Drag Reduction in Two-Phase Annular-Mist Flow of Air and Water

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Two-phase, gas-liquid flow in pipes is frequently encountered and is of significant commercial importance in the petroleum and natural gas industries. The complexity of two-phase momentum transfer problems has led mainly to an accumulation of experimental data and to the analysis of the data by semiempirical extension of single-phase flow models (Govier and Aziz, 1972). The literature in the field is extensive; for example, a general index of 5,232 references on two-phase flow has been compiled by Gouse (1966).

Turbulent flow-drag reduction can be defined as a decrease in the pressure gradient for a given volumetric flow rate. It is usually produced by the addition of a linear, high molecular weight, polymeric material to a low viscosity liquid. Drag reduction has received considerable attention because of the many applications of both theoretical and pragmatic interest. The numerous published studies are almost exclusively concerned with single-phase liquid flow. A number of these single-phase studies are contained in symposium volumes edited by Wells (1969), Savins and Virk (1971), and Sylvester (1973). In addition, excellent reviews of single-phase drag reduction have been published by Patterson et al. (1969), Hoyt (1972), Lumley (1973), and Virk (1975). The mechanism responsible for drag reduction with polymer solutions has not been satisfactorily explained (see, for example, Kumor and Sylvester, 1973; Virk, 1975). The mechanism remains obscure partly because of experimental difficulties (Virk, 1975) and polymer degradation during flow (Sylvester and Kumor, 1973).

Although there have been numeorus studies of drag reduction in single-phase pipe flow of polymer solutions, only four studies of two-phase, gas-liquid, horizontal pipe flow with polymers dissolved in the liquid phase have been published (Oliver and Hoon, 1968; Mahalingam and Valle, 1972; Greskovich and Shrier, 1971; Rosehart et al., 1972). Of these four, all of which were low pressure studies, only two (Greskovich and Shrier, 1971; Rosehart et al., 1972) were concerned with drag reduction. Their experimental work was limited to relatively low gas-liquid ratios (plug or slug flow) not likely to be found in gas production and transportation.

There are a number of potential applications of drag reduction in two-phase flow to gas production and transportation. Examples include the reduction of pumping and compressor requirements in gas-gas condensate pipelines, increased capacity of existing producing equipment, and increased flow from aquifers and other storage facilities.

The purpose of this study was to demonstrate the existence of drag reduction in two-phase, annular-mist flow. Pressure drop data were taken for horizontal flow of air and water in a 1.27 cm diameter pipe at a system pressure of approximately 6.895×10^5 Pa (100 lb/sq in. gauge). Polyethylene oxide, a well-known, water soluble, drag reducing polymer was used. The liquid-gas ratio was varied from 56.2 to 5620 m^3 of liquid per million standard cubic meter of gas (that is, 10 to 1000 barrels of liquid per million standard cubic feet of gas).

EXPERIMENT

The experimental equipment is shown schematically in Figure 1. The air supply was provided by two Ingersoll-Rand compressors and maintained at 9.31×10^5 Pa (135 lb/sq. in. gauge). The air feed line contained an orifice meter run and a pressure regulator which maintained the line pressure at 8.69×10^5 Pa (126 lb/sq in. gauge). The orifice meter monitored the air temperature, the absolute pressure, and the differential pressure drop to provide an accurate and continuous air flow measurement. Water was fed to the system by a FWI triplex pump driven by a variable speed motor. All tubing following the mixing tee was 1.27 cm I.D. stainless steel and was mounted in the same horizontal plane. A calming section of